Math 2210, Spring 2022

Homework 3

Due by 10pm Sep 14

1. **(10 points)** Find a matrix M that induces the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}$$

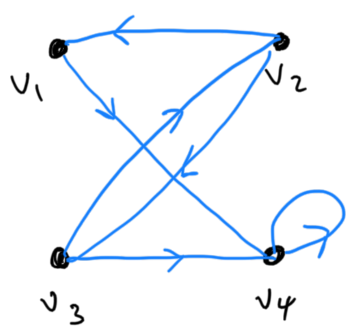
for all x, y. Is T a rotation (what axis and angle?), a reflection (through what?), a combination of these, or not a combination of these? Justify your answer.

2. **(5 + 5 = 10 points)** Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}.$$

(a) Calculate A^2.

In the following diagram, known as a *directed graph*, there is an arrow (called a *directed edge*) from vertex v_j to vertex v_i whenever the (i, j)-entry in A is 1. That is to say, A is the *adjacency matrix* for the directed graph. Another example is presented in the final part of Section 2.3 of Nicholson.



There are two paths of length 2 in our directed graph from v_2 to v_4, namely $v_2 \to v_3 \to v_4$ and $v_2 \to v_1 \to v_4$. Correspondingly, the $(4, 2)$-entry in A^2 is 2.

(b) Explain why this correspondence works for all adjacency matrices. That is, explain why the (i, j)-entry of the square of an adjacency matrix A for a directed graph with vertices $v_1, v_2, \ldots, v_n$ is the number of paths of length 2 from v_j to v_i.

3. **(10 points)** Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

be a 2×2 matrix, where a, b, c, d are real numbers. Show, using row reduction, that if $ad - bc \neq 0$, then A is invertible.